

基于Kane方程的冗余驱动5UPS/PRPU并联机床动力学分析

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摘要:为了解决冗余驱动并联机构的驱动力协调问题,搭建基于动力学的控制系统,实现冗余驱动并联机床的力位混合控制,以原有5UPS/PRPU并联机床为研究对象,改变中间PRPU约束分支为冗余驱动分支,构成冗余驱动并联机床,在其运动学分析的基础上,采用计算效率较高的Kane方法建立了该并联机床的动力学模型;应用ADAMS软件仿真得出驱动力并与Kane动力学模型求解出的驱动力进行比较,验证了Kane动力学模型的正确性,为冗余驱动并联机床力位混合控制研究奠定了理论基础。

关键词: 并联机床; Kane方程; 动力学; 仿真分析

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Dynamics Analysis of 5UPS/PRPU Parallel Machine Tool with Redundant Actuation Based on Kane Equation

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Abstract: In order to further study and solve the problems of the driving force coordination of parallel machine tool with redundant actuation, build a control system based on dynamics and realize the force-position hybrid control, the original 5UPS/PRPU parallel machine tool was taken as research object, and a drive for middle PRPU constraint branch was increased to make it a redundant actuation branch. Finally, the 5UPS/PRPU parallel machine tool with redundant actuation was built. The Kane equation of higher computation efficiency was taken to establish the dynamics model. The mechanical structure was introduced, the kinematics property of the parallel machine tool was analyzed from the three aspects of speed, partial speed and acceleration. The dynamics model of the system was established, and the redundant driving force was solved. By using ADAMS software, the no-load translation example driving force was simulated and compared with the no-load translation example driving force obtained by Kane dynamic model of the parallel machine tool. The results showed that the simulated and calculated driving forces were very similar, the maximum deviation absolute value was only 0.068 N. It proved the validity of Kane dynamical model and obtained the mechanism of driving force coordination and distribution. Redundant actuation can effectively reduce the peak value of driving force of the parallel machine tool and make the driving forces change slowly during the motion process. Besides, it has the advantages of high rigidity, high bearing capacity, good stability and so on. The correct establishment of the dynamic model was the prerequisite of realizing real-time control strategy. It was of great significance to the research on redundant actuation of the parallel machine tool.

Key words: parallel machine tool; Kane equation; dynamics; simulation analysis

引言

并联机器人常用的动力学研究理论有：利用达朗贝尔原理分析机构每个刚体部件的受力状态，列出方程求解每个部件约束反力的 Newton - Euler 法^[1-6]；不出现理想约束反力，不使用动力学函数，只需进行矢量的点积、叉积运算而不需要求导的 Kane 法^[7-12]；消去约束反力，以一种简单的形式建立系统动力学方程的 Lagrange 法^[11-12]；根据作用于理想系统上的主动力在任何虚位移上所做的元功之和为零进行建模的虚功原理法^[13-15]等。对于冗余驱动并联机构的动力学研究，文献[10]运用 Kane 法建立了一种新型冗余驱动 2 自由度摇摆台的动力学模型，文献[16]运用 Newton - Euler 法建立了冗余驱动 3 自由度 Tricept 并联机构的动力学模型，文献[17]运用 Lagrange 法建立了冗余驱动 2PRRR + 1PPRR 3 自由度并联机构的动力学模型，文献[18]基于 Newton - Euler 法建立了一种 3 自由度冗余驱动并联机构的动力学模型等。

目前对于冗余驱动的研究多基于 2、3 自由度冗余驱动并联机构，为了深入研究多自由度冗余驱动并联机构驱动力协调问题，实现基于动力学的力位混合控制，本文采用 Kane 法建立 5 自由度冗余驱动 5UPS/PRPU 并联机床的逆向动力学模型，并用实例对其动力学模型进行仿真。

1 5UPS/PRPU 并联机器人运动学分析

1.1 冗余驱动 5UPS/PRPU 并联机床

冗余驱动 5UPS/PRPU 5 自由度并联机床的结构如图 1 所示，该机构通过 5 个结构完全相同的 UPS(虎克铰、移动副、球副)分支与 1 个 PRPU(移动副、转动副、移动副、虎克铰)分支连接动平台与定平台。6 个分支均为驱动分支，通过控制 5 个 UPS 分支中移动副的伸缩与 PRPU 分支中第 1 个移动副在导轨上的移动来实现动平台的位姿。中间的 PRPU 分支同时又为约束分支，约束动平台使其无法在其轴线方向完成旋转运动，故该并联机床的自由度为 5，形成了具有冗余驱动特性的 5UPS/PRPU 并联机床。该并联机床的机构简图如图 2 所示，在定平台、动平台上分别建立定系 $O_A X_A Y_A Z_A$ 和动系 $O_B X_B Y_B Z_B$ 。

广义坐标选为运动平台位姿表示 $\mathbf{q} = [{}^A X_{B0} \quad {}^A Y_{B0} \quad {}^A Z_{B0} \quad \alpha \quad \beta]^T$ ，则广义速度为广义坐标导数 $\dot{\mathbf{q}} = [{}^A \dot{X}_{B0} \quad {}^A \dot{Y}_{B0} \quad {}^A \dot{Z}_{B0} \quad \dot{\alpha} \quad \dot{\beta}]^T$ ，广义速率取 $\mathbf{u} = \dot{\mathbf{q}} = [{}^A \dot{X}_{B0} \quad {}^A \dot{Y}_{B0} \quad {}^A \dot{Z}_{B0} \quad \dot{\alpha} \quad \dot{\beta}]^T$ ，运动平台的线加

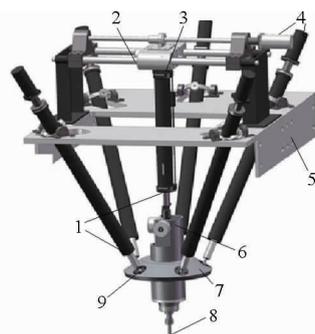


图 1 5UPS/PRPU 机床模型结构图

Fig. 1 Virtual prototype of 5UPS/PRPU parallel machine tool
1. 移动副 3. 转动副 4. 电动机 5. 定平台 6. 虎克铰
7. 动平台 8. 主轴 9. 球副

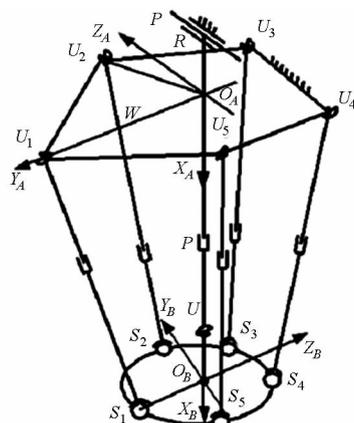


图 2 5UPS/PRPU 机床结构简图

Fig. 2 Schematic diagram of 5UPS/PRPU parallel machine tool

速度可表示为 ${}^A \mathbf{a}_B = [{}^A \ddot{X}_{B0} \quad {}^A \ddot{Y}_{B0} \quad {}^A \ddot{Z}_{B0}]^T$ ，这些均作为 5UPS/PRPU 冗余驱动并联机床进行动力学建模分析的已知量。

1.2 速度分析

1.2.1 运动平台速度分析

本文运动平台的姿态用 $Z - Y - X$ 欧拉角 (α, β, γ) 来表示，其导数 $\dot{\alpha}$ 位于 Z 轴， $\dot{\beta}$ 位于 Y' 轴， $\dot{\gamma}$ 位于 X'' 轴，而 $\dot{\alpha}, \dot{\beta}$ 和 $\dot{\gamma}$ 并不相互垂直，将 $\dot{\alpha}, \dot{\beta}$ 和 $\dot{\gamma}$ 转换到定系得到运动平台的角速度

$${}^A \boldsymbol{\omega}_B = \mathbf{R}(Z, \alpha) \begin{bmatrix} 0 \\ 0 \\ \dot{\alpha} \end{bmatrix} + \mathbf{R}(Z, \alpha) \mathbf{R}(Y, \beta) \begin{bmatrix} 0 \\ \dot{\beta} \\ 0 \end{bmatrix} + \mathbf{R}(Z, \alpha) \mathbf{R}(Y, \beta) \mathbf{R}(X, \gamma) \begin{bmatrix} \dot{\gamma} \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

本机床中 $\dot{\gamma} = 0$ ，所以式(1)变为

$${}^A \boldsymbol{\omega}_B = \begin{bmatrix} 0 & -\sin\alpha \\ 0 & \cos\alpha \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix} \quad (2)$$

则运动平台的速度表示为

$$\begin{cases} {}^A\mathbf{V}_{BO} = \mathbf{V}u \\ {}^A\boldsymbol{\omega}_{BO} = \mathbf{W}u \end{cases} \quad (3)$$

其中

$$\mathbf{V} = \begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{O}_{3 \times 2} \\ \mathbf{O}_{3 \times 3} & \begin{bmatrix} 0 & -\sin\alpha \\ 0 & \cos\alpha \\ 1 & 0 \end{bmatrix} \end{bmatrix}$$

式中 $\mathbf{I}_{m \times n}$ —— $m \times n$ 阶单位矩阵

$\mathbf{O}_{m \times n}$ —— $m \times n$ 阶零矩阵

1.2.2 UPS 分支速度分析

UPS 分支分为两部分:上半部分为摆杆,通过 U 副铰接在固定平台上,其铰点为 $U_i (i=1,2,\dots,5)$; 下半部分为伸杆,通过 S 副铰接在运动平台上,其铰点为 $S_i (i=1,2,\dots,5)$ 。 ${}^A\mathbf{U}_i$ 为由 O_A 指向 U_i 的向量在定系 A 中的表示, ${}^A\mathbf{r}_{S_i}$ 为由 O_B 指向 S_i 的向量在定系 A 中的表示, ${}^A\mathbf{P}_{BO}$ 为 O_B 点在定系内的坐标。

UPS 分支杆长

$$l_i = |{}^A\mathbf{P}_{BO} + {}^A\mathbf{r}_{S_i} - {}^A\mathbf{U}_i| \quad (4)$$

UPS 分支单位向量 ${}^A\mathbf{n}_i$ 为

$${}^A\mathbf{n}_i = \frac{{}^A\mathbf{P}_{BO} + {}^A\mathbf{r}_{S_i} - {}^A\mathbf{U}_i}{|{}^A\mathbf{P}_{BO} + {}^A\mathbf{r}_{S_i} - {}^A\mathbf{U}_i|} \quad (5)$$

由速度合成定理可得 ${}^A\mathbf{V}_{S_i}$ 的 2 种表示

$${}^A\mathbf{V}_{S_i} = {}^A\mathbf{V}_{BO} + {}^A\boldsymbol{\omega}_B \times {}^A\mathbf{r}_{S_i} \quad (6)$$

$${}^A\mathbf{V}_{S_i} = {}^A\boldsymbol{\omega}_i \times {}^A\mathbf{n}_i l_i + {}^A\mathbf{n}_i \dot{l}_i \quad (7)$$

式中 \dot{l}_i —— 伸杆沿摆杆轴线方向速率,由式(4)求得得到

式(7)两边分别左叉乘分支单位向量 ${}^A\mathbf{n}_i$, 因为 ${}^A\mathbf{n}_i$ 与 ${}^A\boldsymbol{\omega}_i$ 近似垂直,化简得到

$${}^A\boldsymbol{\omega}_i = \frac{{}^A\mathbf{n}_i \times {}^A\mathbf{V}_{S_i}}{l_i} \quad (8)$$

联立式(3)、(6)、(8),化简得到

$${}^A\boldsymbol{\omega}_i = \frac{1}{l_i} {}^A\hat{\mathbf{n}}_i (\mathbf{V} - {}^A\hat{\mathbf{r}}_{S_i} \mathbf{W}) u \quad (9)$$

式中 ${}^A\hat{\mathbf{n}}_i$ —— ${}^A\mathbf{n}_i$ 的反对称矩阵

${}^A\hat{\mathbf{r}}_{S_i}$ —— ${}^A\mathbf{r}_{S_i}$ 的反对称矩阵

设摆杆质心到 U 副的距离为 l_{i1} , 则可得摆杆质心在定系 A 中的线速度为

$${}^A\mathbf{V}_{i1} = {}^A\boldsymbol{\omega}_i \times {}^A\mathbf{n}_i l_{i1} \quad (10)$$

联立式(9)、(10),化简得到

$${}^A\mathbf{V}_{i1} = -\frac{l_{i1}}{l_i} {}^A\hat{\mathbf{n}}_i^2 (\mathbf{V} - {}^A\hat{\mathbf{r}}_{S_i} \mathbf{W}) u \quad (11)$$

设伸杆质心到 U 副的距离为 l_{i2} , 则可得到伸杆的质心在定系 A 中的线速度为

$${}^A\mathbf{V}_{i2} = {}^A\boldsymbol{\omega}_i \times {}^A\mathbf{n}_i l_{i2} + {}^A\mathbf{n}_i ({}^A\mathbf{n}_i^T {}^A\mathbf{V}_{S_i}) \quad (12)$$

联立式(3)、(6)、(9)、(12),化简得到

$${}^A\mathbf{V}_{i2} = \left(-\frac{l_{i2}}{l_i} {}^A\hat{\mathbf{n}}_i^2 + {}^A\mathbf{n}_i {}^A\mathbf{n}_i^T \right) (\mathbf{V} - {}^A\hat{\mathbf{r}}_{S_i} \mathbf{W}) u \quad (13)$$

1.2.3 PRPU 分支速度分析

动系 B 相对于定系 A 的旋转变换矩阵为

$${}^A\mathbf{R}_B = {}^A\mathbf{R}_{Z-Y-X}(\alpha, \beta, \gamma) \quad (14)$$

根据 Z-Y-X 欧拉角定义,由式(14)进一步得到

$${}^A\mathbf{R}_B = \mathbf{R}(Z, \alpha) \mathbf{R}(Y, \beta) \mathbf{R}(X, \gamma = 90^\circ) = \begin{bmatrix} \cos\alpha \cos\beta & \cos\alpha \sin\beta & \sin\alpha \\ \sin\alpha \cos\beta & \sin\alpha \sin\beta & -\cos\alpha \\ -\sin\beta & \cos\beta & 0 \end{bmatrix} \quad (15)$$

从中间分支 U 副中心在定系 A 中的位置向量可表示为

$${}^A\mathbf{P}_U = {}^A\mathbf{P}_{BO} + {}^A\mathbf{R}_B {}^B\mathbf{P}_U \quad (16)$$

式中 ${}^B\mathbf{P}_U$ —— 中间分支 U 副中心在动系 B 中的位置向量

对中间分支构建局部坐标系,如图3所示,坐标系 O 的原点位于定系 A 的 X 轴与滚珠丝杠中心线的交点处,坐标系 1 的原点位于滚珠螺母的中心。

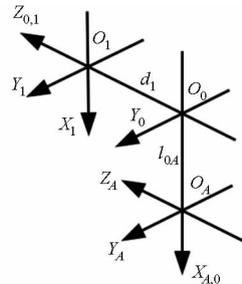


图3 PRPU 分支局部坐标系

Fig.3 Local coordinate system of PRPU branch

由图3容易得到 PRPU 分支第 1 个 P 副中心(坐标系 1 原点)在定系 A 中的位置向量

$${}^A\mathbf{P}_1 = [-l_{0A} \quad 0 \quad d_1]^T \quad (17)$$

则该分支摆杆单位向量为

$${}^A\mathbf{n}'_2 = \frac{{}^A\mathbf{P}_{BO} + {}^A\mathbf{R}_B {}^B\mathbf{P}_U - {}^A\mathbf{P}_1}{|{}^A\mathbf{P}_{BO} + {}^A\mathbf{R}_B {}^B\mathbf{P}_U - {}^A\mathbf{P}_1|} \quad (18)$$

摆杆杆长

$$l'_2 = |{}^A\mathbf{P}_{BO} + {}^A\mathbf{R}_B {}^B\mathbf{P}_U - {}^A\mathbf{P}_1| \quad (19)$$

由速度合成定理可得到 PRPU 分支 U 副中心的线速度的 2 种表示

$${}^A\mathbf{V}_U = {}^A\mathbf{V}_{BO} + {}^A\boldsymbol{\omega}_B \times {}^A\mathbf{r}_U \quad (20)$$

$${}^A\mathbf{V}_U = {}^A\mathbf{V}'_1 + {}^A\boldsymbol{\omega}'_2 \times {}^A\mathbf{n}'_2 l'_2 + {}^A\mathbf{n}'_2 \dot{l}'_2 \quad (21)$$

式中 ${}^A\mathbf{V}'_1$ —— PRPU 分支第一个移动滑块线速度

${}^A\boldsymbol{\omega}'_2$ —— PRPU 分支摆杆角速度

${}^A\mathbf{r}_U$ —— O_B 指向 U 副中心的向量在定系 A 中的表示

对中间分支分析可知,U 副中心线速度在定系

A 中的 Z 方向分量来自第一个移动滑块的运动 ${}^A\mathbf{V}'_1$, 而其在 XY 方向上的分量完全来自摆杆的摆动 ${}^A\boldsymbol{\omega}'_2$, 所以分别取式(20)和式(21)的 Z 方向分量和 XY 方向分量同时联立式(3), 化简得到

$$\begin{cases} {}^A\mathbf{V}'_1 = \mathbf{H}_Z (\mathbf{V} - {}^A\hat{\mathbf{r}}_U \mathbf{W}) u \\ {}^A\boldsymbol{\omega}'_2 = \frac{1}{l'_2} {}^A\hat{\mathbf{n}}'_2 \mathbf{H}_{XY} (\mathbf{V} - {}^A\hat{\mathbf{r}}_U \mathbf{W}) u \end{cases} \quad (22)$$

$$\text{其中 } \mathbf{H}_Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{H}_{XY} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

设中间分支摆杆摆动件质心到摆杆顶端的距离为 l'_{21} , 则其质心线速度为

$${}^A\mathbf{V}'_{21} = {}^A\mathbf{V}'_1 + {}^A\boldsymbol{\omega}'_2 \times {}^A\mathbf{n}'_2 l'_{21} \quad (23)$$

联立式(22)、(23), 化简得到

$${}^A\mathbf{V}'_{21} = \left(\mathbf{H}_Z - \frac{l'_{21} {}^A\hat{\mathbf{n}}'_2 {}^A\hat{\mathbf{n}}'_2{}^T}{l'_2} \mathbf{H}_{XY} \right) (\mathbf{V} - {}^A\hat{\mathbf{r}}_U \mathbf{W}) u \quad (24)$$

设 PRPU 分支摆杆伸缩件质心到摆杆顶的距离为 l'_{22} , 则其质心线速度为

$${}^A\mathbf{V}'_{22} = {}^A\mathbf{V}'_{21} + {}^A\boldsymbol{\omega}'_2 \times {}^A\mathbf{n}'_2 l'_{22} + {}^A\mathbf{n}'_2 ({}^A\mathbf{n}'_2{}^T \mathbf{V}_U) \quad (25)$$

联立式(3)、(20)、(22), 化简得到

$${}^A\mathbf{V}'_{22} = \left(\mathbf{H}_Z - \frac{l'_{22} {}^A\hat{\mathbf{n}}'_2 {}^A\hat{\mathbf{n}}'_2{}^T}{l'_2} \mathbf{H}_{XY} + {}^A\mathbf{n}'_2 {}^A\mathbf{n}'_2{}^T \right) \times (\mathbf{V} - {}^A\hat{\mathbf{r}}_U \mathbf{W}) u \quad (26)$$

1.3 偏速度分析

1.3.1 运动平台偏速度分析

由式(3)容易得到运动平台的偏线速度和偏角速度为

$$\begin{cases} {}^* \mathbf{V}_B = \mathbf{V} \\ {}^* \boldsymbol{\omega}_B = \mathbf{W} \end{cases} \quad (27)$$

1.3.2 UPS 分支偏速度分析

由式(9)、(11)容易得到 UPS 分支摆杆质心的偏线速度和偏角速度为

$$\begin{cases} {}^* \mathbf{V}_{i1} = -\frac{l_{i1} {}^A\hat{\mathbf{n}}'_i {}^A\hat{\mathbf{n}}'_i{}^T}{l_i} (\mathbf{V} - {}^A\hat{\mathbf{r}}_{S_i} \mathbf{W}) \\ {}^* \boldsymbol{\omega}_{i1} = \frac{1}{l_i} {}^A\hat{\mathbf{n}}'_i (\mathbf{V} - {}^A\hat{\mathbf{r}}_{S_i} \mathbf{W}) \end{cases} \quad (28)$$

由式(9)、(13)容易得到 UPS 分支伸杆质心的偏线速度和偏角速度为

$$\begin{cases} {}^* \mathbf{V}_{i2} = \left(-\frac{l_{i2} {}^A\hat{\mathbf{n}}'_i {}^A\hat{\mathbf{n}}'_i{}^T}{l_i} + {}^A\mathbf{n}_i {}^A\mathbf{n}_i{}^T \right) (\mathbf{V} - {}^A\hat{\mathbf{r}}_{S_i} \mathbf{W}) \\ {}^* \boldsymbol{\omega}_{i2} = \frac{1}{l_i} {}^A\hat{\mathbf{n}}'_i (\mathbf{V} - {}^A\hat{\mathbf{r}}_{S_i} \mathbf{W}) \end{cases} \quad (29)$$

1.3.3 PRPU 分支偏速度分析

由式(22)容易得到该分支第一个移动滑块的偏线速度和偏角速度为

$$\begin{cases} {}^* \mathbf{V}'_1 = \mathbf{H}_Z (\mathbf{V} - {}^A\hat{\mathbf{r}}_{S_i} \mathbf{W}) \\ {}^* \boldsymbol{\omega}'_1 = \mathbf{O}_{3 \times 3} \end{cases} \quad (30)$$

由式(22)、(24)得到该分支摆杆摆动件质心的偏线速度和偏角速度为

$$\begin{cases} {}^* \mathbf{V}'_{21} = \left(\mathbf{H}_Z - \frac{l'_{21} {}^A\hat{\mathbf{n}}'_2 {}^A\hat{\mathbf{n}}'_2{}^T}{l'_2} \mathbf{H}_{XY} \right) (\mathbf{V} - {}^A\hat{\mathbf{r}}_U \mathbf{W}) \\ {}^* \boldsymbol{\omega}'_{21} = \frac{1}{l'_2} {}^A\hat{\mathbf{n}}'_2 \mathbf{H}_{XY} (\mathbf{V} - {}^A\hat{\mathbf{r}}_U \mathbf{W}) \end{cases} \quad (31)$$

由式(22)、(26)得到该分支摆杆伸缩件质心的偏线速度和偏角速度为

$$\begin{cases} {}^* \mathbf{V}'_{22} = \left(\mathbf{H}_Z - \frac{l'_{22} {}^A\hat{\mathbf{n}}'_2 {}^A\hat{\mathbf{n}}'_2{}^T}{l'_2} \mathbf{H}_{XY} + {}^A\mathbf{n}'_2 {}^A\mathbf{n}'_2{}^T \right) (\mathbf{V} - {}^A\hat{\mathbf{r}}_U \mathbf{W}) \\ {}^* \boldsymbol{\omega}'_{22} = \frac{1}{l'_2} {}^A\hat{\mathbf{n}}'_2 \mathbf{H}_{XY} (\mathbf{V} - {}^A\hat{\mathbf{r}}_U \mathbf{W}) \end{cases} \quad (32)$$

1.4 加速度分析

1.4.1 运动平台加速度分析

由已知条件得到运动平台的线加速度, 对式(2)求导得其角加速度

$$\begin{cases} {}^A \mathbf{a}_B = [{}^A \ddot{X}_{B0} \quad {}^A \ddot{Y}_{B0} \quad {}^A \ddot{Z}_{B0}]^T \\ {}^A \boldsymbol{\varepsilon}_B = \begin{bmatrix} -\cos\alpha \ddot{\alpha} - \sin\alpha \dot{\alpha}^2 \\ -\sin\alpha \ddot{\alpha} + \cos\alpha \dot{\alpha}^2 \\ \ddot{\alpha} \end{bmatrix} \end{cases} \quad (33)$$

1.4.2 UPS 分支构件加速度分析

根据加速度合成定理得 S_i 点线加速度为

$${}^A \mathbf{a}_{S_i} = {}^A \mathbf{a}_B + {}^A \boldsymbol{\varepsilon}_B \times {}^A \mathbf{r}_{S_i} \quad (34)$$

对 UPS 分支, 忽略牵连加速度中的法向加速度, 得到 S 副铰点 S_i 的加速度为

$${}^A \mathbf{a}_{S_i} = {}^A \mathbf{a}_{S_i}^e + {}^A \mathbf{a}_{S_i}^x + {}^A \mathbf{a}_{S_i}^c \quad (35)$$

其中

$${}^A \mathbf{a}_{S_i}^e = {}^A \boldsymbol{\varepsilon}_i \times {}^A \mathbf{n}_i l_i \quad {}^A \mathbf{a}_{S_i}^x = {}^A \mathbf{n}_i \ddot{l}_i$$

$${}^A \mathbf{a}_{S_i}^c = 2 {}^A \boldsymbol{\omega}_i \times {}^A \mathbf{n}_i \dot{l}_i$$

式中 ${}^A \mathbf{a}_{S_i}^e$ ——牵连加速度 ${}^A \mathbf{a}_{S_i}^x$ ——相对加速度

${}^A \mathbf{a}_{S_i}^c$ ——科氏加速度

为减少求导运算, 将式(35)中的相对加速度和科氏加速度作如下简化

$${}^A \mathbf{a}_{S_i}^x = {}^A \mathbf{n}_i \ddot{l}_i = {}^A \mathbf{n}_i {}^A \mathbf{n}_i{}^T \mathbf{a}_{S_i} \quad (36)$$

$${}^A \mathbf{a}_{S_i}^c = 2 {}^A \boldsymbol{\omega}_i \times {}^A \mathbf{n}_i \dot{l}_i = 2 {}^A \boldsymbol{\omega}_i \times ({}^A \mathbf{n}_i {}^A \mathbf{n}_i{}^T \mathbf{V}_{S_i}) \quad (37)$$

联立式(34)、(35)两边左叉乘 ${}^A \mathbf{n}_i$, 将式(37)代入, 得到 UPS 分支角加速度

$${}^A \boldsymbol{\varepsilon}_i = \frac{1}{l_i} \left\{ {}^A \mathbf{n}_i \times ({}^A \mathbf{a}_B + {}^A \mathbf{n}_i \times ({}^A \boldsymbol{\varepsilon}_B \times {}^A \mathbf{r}_{S_i})) - 2 {}^A \mathbf{n}_i \times [{}^A \boldsymbol{\omega}_i \times ({}^A \mathbf{n}_i {}^A \mathbf{n}_i{}^T \mathbf{V}_{S_i})] \right\} \quad (38)$$

UPS 分支摆杆质心的线加速度为

$${}^A \mathbf{a}_{i1} = {}^A \boldsymbol{\varepsilon}_i \times {}^A \mathbf{n}_i l_{i1} \quad (39)$$

UPS 分支伸杆质心的线加速度为

$${}^A \mathbf{a}_{i2} = {}^A \boldsymbol{\varepsilon}_i \times {}^A \mathbf{n}_i l_{i2} + {}^A \mathbf{n}_i {}^A \mathbf{n}_i^{\text{T}A} \mathbf{a}_{S_i} + 2 {}^A \boldsymbol{\omega}_i \times ({}^A \mathbf{n}_i {}^A \mathbf{n}_i^{\text{T}A} \mathbf{V}_{S_i}) \quad (40)$$

1.4.3 PRPU 分支构件加速度分析

运动平台 U 副铰点的线加速度为

$${}^A \mathbf{a}_U = {}^A \mathbf{a}_B + {}^A \boldsymbol{\varepsilon}_B \times {}^A \mathbf{r}_U \quad (41)$$

运动平台 U 副铰点的线加速度还可表示为

$${}^A \mathbf{a}_U = {}^A \mathbf{a}'_1 + {}^A \boldsymbol{\varepsilon}'_2 \times {}^A \mathbf{n}'_2 l'_2 + {}^A \mathbf{n}'_2 \dot{l}'_2 + 2 {}^A \boldsymbol{\omega}'_2 \times {}^A \mathbf{n}'_2 \dot{l}'_2 \quad (42)$$

同式(22)可得

$$\begin{cases} {}^A \mathbf{a}'_1 = \mathbf{H}_Z ({}^A \mathbf{a}_B + {}^A \boldsymbol{\varepsilon}_B \times {}^A \mathbf{r}_U) \\ {}^A \boldsymbol{\varepsilon}'_2 \times {}^A \mathbf{n}'_2 l'_2 + 2 {}^A \boldsymbol{\omega}'_2 \times {}^A \mathbf{n}'_2 \dot{l}'_2 + {}^A \mathbf{n}'_2 \ddot{l}'_2 = \\ \mathbf{H}_{XY} ({}^A \mathbf{a}_B + {}^A \boldsymbol{\varepsilon}_B \times {}^A \mathbf{r}_U) \end{cases} \quad (43)$$

对式(43)中的第2式两边同时左叉乘 ${}^A \mathbf{n}'_2$ 化简得

$${}^A \boldsymbol{\varepsilon}'_2 = \frac{1}{l'_2} [{}^A \mathbf{n}'_2 \times \mathbf{H}_{XY} ({}^A \mathbf{a}_B + {}^A \boldsymbol{\varepsilon}_B \times {}^A \mathbf{r}_U) - 2 {}^A \hat{\mathbf{n}}'_2 \times ({}^A \boldsymbol{\omega}'_2 \times {}^A \mathbf{n}'_2 \dot{l}'_2)] \quad (44)$$

化简式(44)中的导数项得到 PRPU 分支摆杆的角加速度

$${}^A \boldsymbol{\varepsilon}'_2 = \frac{1}{l'_2} [{}^A \mathbf{n}'_2 \times \mathbf{H}_{XY} ({}^A \mathbf{a}_B + {}^A \boldsymbol{\varepsilon}_B \times {}^A \mathbf{r}_U) - 2 {}^A \hat{\mathbf{n}}'_2 \times \hat{\boldsymbol{\omega}}'_2 ({}^A \mathbf{n}'_2 {}^A \mathbf{n}'_2^{\text{T}A} \mathbf{V}_U)] \quad (45)$$

由式(43)的第1式可得 PRPU 分支第一个移动滑块的线加速度为

$${}^A \mathbf{a}'_1 = \mathbf{H}_Z ({}^A \mathbf{a}_B + {}^A \boldsymbol{\varepsilon}_B \times {}^A \mathbf{r}_U) \quad (46)$$

分析可知该滑块并无转动自由度,则其角加速度为

$${}^A \boldsymbol{\varepsilon}'_1 = \mathbf{O}_{3 \times 1} \quad (47)$$

PRPU 分支摆杆摆动作件质心的线加速度为

$${}^A \mathbf{a}'_{21} = {}^A \mathbf{a}'_1 + {}^A \boldsymbol{\varepsilon}'_2 \times {}^A \mathbf{n}'_2 l'_{21} \quad (48)$$

PRPU 分支摆杆伸缩件质心的线加速度为

$${}^A \mathbf{a}'_{22} = {}^A \mathbf{a}'_1 + {}^A \boldsymbol{\varepsilon}'_2 \times {}^A \mathbf{n}'_2 l'_{22} + 2 {}^A \boldsymbol{\omega}'_2 \times {}^A \mathbf{n}'_2 \dot{l}'_2 + {}^A \mathbf{n}'_2 \ddot{l}'_2 \quad (49)$$

化简式(49)中的导数项得

$${}^A \mathbf{a}'_{22} = {}^A \mathbf{a}'_1 + {}^A \boldsymbol{\varepsilon}'_2 \times {}^A \mathbf{n}'_2 l'_{22} + 2 {}^A \boldsymbol{\omega}'_2 \times ({}^A \mathbf{n}'_2 {}^A \mathbf{n}'_2^{\text{T}A} \mathbf{V}_U) + {}^A \mathbf{n}'_2 {}^A \mathbf{n}'_2^{\text{T}A} \mathbf{a}_U \quad (50)$$

2 动力学分析

冗余系统广义主动力为

$$\begin{aligned} \mathbf{F}_{zr} = m_B \mathbf{g}^* \mathbf{v}_B + \sum_{i=1}^2 m'_{2i} \mathbf{g}^* \mathbf{v}'_{2i} + \\ \sum_{i=1}^5 (m_{i1} \mathbf{g} - \mathbf{f}_i^A \mathbf{n}_i)^* \mathbf{v}_{i1} + \\ \sum_{i=1}^5 (m_{i2} \mathbf{g} + \mathbf{f}_i^A \mathbf{n}_i)^* \mathbf{v}_{i2} + \mathbf{f}_6^A \mathbf{n}'_1^* \mathbf{v}'_1 \end{aligned} \quad (51)$$

其中 ${}^A \mathbf{n}'_1 = [0 \ 0 \ 1]^{\text{T}}$

式中 ${}^A \mathbf{n}'_1$ ——冗余驱动力 \mathbf{f}_6 的单位向量
冗余系统广义惯性力为

$$\begin{aligned} {}^* \mathbf{F}_{zr} = -m_B {}^A \mathbf{a}_B^* \mathbf{v}_B - \sum_{i=1}^2 m'_{2i} {}^A \mathbf{a}_{2i}^* \mathbf{v}'_{2i} - \\ \sum_{i=1}^5 m_{i1} {}^A \mathbf{a}_{i1}^* \mathbf{v}_{i1} - \sum_{i=1}^5 m_{i2} {}^A \mathbf{a}_{i2}^* \mathbf{v}_{i2} - m_1^A \mathbf{a}'_1^* \mathbf{v}'_1 - \\ [{}^I_B {}^A \boldsymbol{\varepsilon}_B + {}^A \boldsymbol{\omega}_i \times ({}^I_B {}^A \boldsymbol{\omega}_B)]^* \boldsymbol{\omega}_B - \\ \sum_{i=1}^5 [{}^I_U {}^A \boldsymbol{\varepsilon}_i + {}^A \boldsymbol{\omega}_i \times ({}^I_U {}^A \boldsymbol{\omega}_i)]^* \boldsymbol{\omega}_i - \\ \sum_{i=1}^5 [{}^I_S {}^A \boldsymbol{\varepsilon}_i + {}^A \boldsymbol{\omega}_i \times ({}^I_S {}^A \boldsymbol{\omega}_i)]^* \boldsymbol{\omega}_i - \\ \sum_{i=1}^2 [{}^I_i {}^A \boldsymbol{\varepsilon}'_i + {}^A \boldsymbol{\omega}'_i \times ({}^I_i {}^A \boldsymbol{\omega}'_i)]^* \boldsymbol{\omega}'_i \end{aligned} \quad (52)$$

则 Kane 动力学模型为

$$\mathbf{F}_{zr} + {}^* \mathbf{F}_{zr} = 0 \quad (53)$$

引入 \mathbf{F}'_{zr} , 令

$$\begin{aligned} \mathbf{F}'_{zr} = \mathbf{F}_{zr} - m_B \mathbf{g}^* \mathbf{v}_B - \sum_{i=1}^2 m'_{2i} \mathbf{g}^* \mathbf{v}'_{2i} - \\ \sum_{i=1}^5 m_{i1} \mathbf{g}^* \mathbf{v}_{i1} - \sum_{i=1}^5 m_{i2} \mathbf{g}^* \mathbf{v}_{i2} \end{aligned} \quad (54)$$

则式(53)化简为

$$\mathbf{J}_r \mathbf{F}_{qr} = -\mathbf{F}'_{zr} - {}^* \mathbf{F}_{zr} \quad (55)$$

其中 $\mathbf{F}_{qr} = [\mathbf{f}_1 \ \mathbf{f}_2 \ \mathbf{f}_3 \ \mathbf{f}_4 \ \mathbf{f}_5 \ \mathbf{f}_6]^{\text{T}}$

$$\mathbf{J}_r = \begin{bmatrix} {}^A \mathbf{n}_1^{\text{T}} (* \mathbf{v}_{11} - * \mathbf{v}_{12}) \\ {}^A \mathbf{n}_2^{\text{T}} (* \mathbf{v}_{21} - * \mathbf{v}_{22}) \\ {}^A \mathbf{n}_3^{\text{T}} (* \mathbf{v}_{31} - * \mathbf{v}_{32}) \\ {}^A \mathbf{n}_4^{\text{T}} (* \mathbf{v}_{41} - * \mathbf{v}_{42}) \\ {}^A \mathbf{n}_5^{\text{T}} (* \mathbf{v}_{51} - * \mathbf{v}_{52}) \\ {}^A \mathbf{n}'_1^{\text{T}} * \mathbf{v}'_1 \end{bmatrix}_{5 \times 6}$$

式中 \mathbf{F}_{qr} ——冗余驱动力矢

\mathbf{J}_r ——传力矩阵

本文以驱动力 2 范数最小为优化目标,求得 5UPS-PRPU 冗余并联机床的驱动力矢为

$$\mathbf{F}_{qr} = \mathbf{J}_r^{\text{T}} (\mathbf{J}_r \mathbf{J}_r^{\text{T}})^{-1} (-\mathbf{F}'_{zr} - {}^* \mathbf{F}_{zr}) \quad (56)$$

同理可求得机床在非冗余时的驱动力矢为

$$\mathbf{F}_{qf} = \mathbf{J}_f^{-1} (-\mathbf{F}'_{zf} - {}^* \mathbf{F}_{zf}) \quad (57)$$

3 Kane 动力学模型验证

由于存在多种冗余驱动力优化算法,因此本文不验证某种优化算法得到的冗余驱动力,而验证机床非冗余驱动力,即可验证 Kane 动力学模型的正确性。将 ADAMS 仿真得出的驱动力与 Kane 动力学模型求解出的驱动力进行比较,若两者相等或相差很小,则认为所建立的 Kane 动力学模型正确。

并联机床常见的运动形式是平动,因此本文给

出了 5UPS/PRPU 冗余并联机床动力学仿真的空载平动算例。采用三维实体模型中的机床结构参数,机床的运动参数如表 1 所示。运动平台中心的轨迹参数方程为

$$\begin{cases} x = x_0 \\ y = R_0 \sin \omega t \\ z = R_0 \cos \omega t \end{cases} \quad (58)$$

式中 x_0 ——轨迹所在平面的 X 坐标
 R_0 ——轨迹半径 ω ——轨迹角速度
 t ——运动时间

表 1 仿真参数
 Tab. 1 Simulation parameters

参数	$\alpha/$ rad	$\beta/$ rad	$\gamma/$ rad	$\omega/$ (rad·s ⁻¹)	R_0/m	x_0/m
数值	0	0	$\pi/2$	0.179	0.15	1.019

将上述平动算例代入本文所建立的机床空载非冗余动力学模型,求得该机床在一个圆周运动周期内 5 个驱动分支的驱动力如图 4 所示,由图可以看出,在走圆过程中由非冗余动力学模型计算出的驱动力都呈正、余弦曲线变化,这与机床实际运行过程中各个轴的运动是正余弦函数相一致,可初步判定

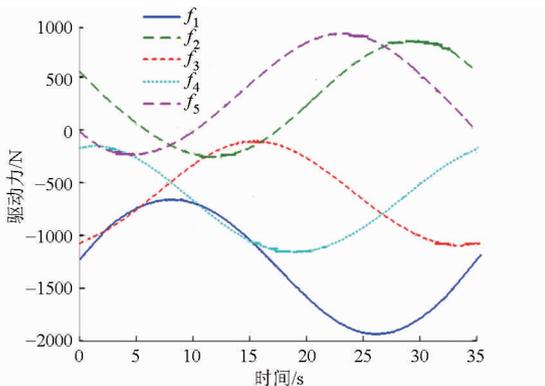


图 4 非冗余动力学模型平动算例驱动力
 Fig. 4 Driving forces of non-redundant dynamic model translation example

动力学模型的正确性。

在 ADAMS 中对非冗余机床平动算例进行动力学仿真,仿真参数不变(表 1),得出 5 个分支杆的驱动力数据,即驱动力测量值。将由动力学模型求解出的驱动力理论值与 ADAMS 得到的测量值进行比较,得出如图 5 所示的驱动力偏差曲线,从图中看出,两组驱动力的偏差很小,偏差绝对值的最大值为 0.068 N,可以认为数值基本相等,从而可以验证所建立的 Kane 动力学模型的正确性。

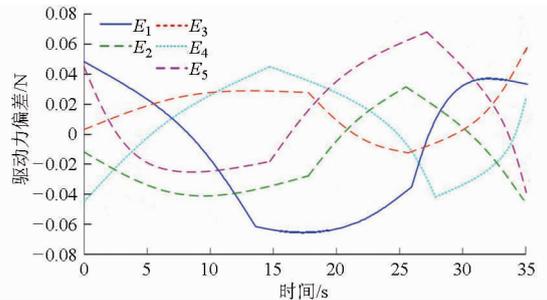


图 5 非冗余动力学模型平动算例的理论驱动力与测量驱动力偏差

Fig. 5 Deviations between theoretical driving force and measured driving force of non-redundant dynamic model translation example

4 结论

(1) 根据冗余驱动并联机床机构的构型特点,改变原中间 PRPU 被动约束分支为主动冗余驱动分支,构成冗余驱动并联机床,采用计算效率较高的 Kane 方法建立了该多自由度冗余驱动并联机器人的动力学模型,得到了驱动力协调分配机理。

(2) 运用 ADAMS 软件仿真得的驱动力,与 Kane 动力学模型求解出的驱动力进行比较,两组驱动力的偏差很小,验证了 Kane 动力学模型的正确性,为该机床后续的驱动力协调问题及基于动力学的力位混合控制研究奠定了理论基础。

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